Optical Bistability in a Nonlinear Resonator With Saturable Losses and Intensity-Dependent Refractive Index

Swati Bhargava, Claudio Porzi, Prasanta Kumar Datta, Antonella Bogoni, Luca Potì, and Ranjan Gangopadhyay

Abstract—We present numerical results relative to a bistable optical device comprised of an asymmetric Fabry-Perot cavity embedding a nonlinear optical medium. Both the real and imaginary part of the third order nonlinear susceptibility are considered in our calculations. Optimized device operation, expressed in terms of the contrast ratio between the two stable states, is then discussed as a function of the different cavity parameters. The presented method can be a useful tool for designing practical devices with application in the field of all-optical switching, optical regeneration and optical signal processing.

Index Terms—Optical bistability, nonlinear optical cavities, semiconductor optical nonlinearities, all-optical signal processing

I. INTRODUCTION

Optical bistable/multistable devices have a large number of applications in the field of optical communications. Optical bistability can be exploited to realize all-optical switches, logic gates, and memory elements such as optical flip-flops. A system is said to be bistable (or multistable) when it can assume two (or more than two) output stable states for the same value of an applied input signal over some range of input signal values. The state assumed depends on the history of the input. Optical bistability requires a nonlinear medium in presence of feedback. When the nonlinearities are optically-induced, and the feedback is also provided in the optical domain, an all-optical bistable device can be fabricated. For practical application in a real system, an optical bistable device should offer a high contrast-ratio between the two stable states, a low switching energy and a fair throughput, compactness and easiness of fabrication. Historically, Fabry-Perot cavities filled with a nonlinear medium have been first proposed and demonstrated to provide optical bistability [1]-[5]. The nonlinear medium could be either a dispersive one (i.e., with an intensity dependent refractive index) [1]-[3], or an absorptive one (i.e., with losses that saturate with intensity) [4], [5]. Theoretical analysis aimed to optimize device operation have been presented for both the dispersive and absorptive nonlinear cavity. However, to our knowledge, no work including simultaneously both the effects of saturating losses and intensity-dependent variation of the refractive index has been discussed. Here we present the analysis of a nonlinear cavity filled with an absorptive and dispersive medium, designed to operate as an efficient, high-contrast optical bistable device. The dispersive and absorptive nonlinearities can be provided for instance by standard semiconductor multiple-quantum wells. We restrict ourselves to the case of a cavity operating in reflection, with a rear mirror having an ideal intensity reflectivity of 100%, and a partially transmitting front mirror. This configuration has been demonstrated to offer advantages in terms of reduced switching energy and maximized throughput [2], [3]. These kinds of devices are usually called asymmetric Fabry-Perot (AFP) cavities. Dispersive optical bistability has also been demonstrated in microring resonators [6], [7], fiber Bragg gratings [8], or photonic crystals devices [9]. Although in most of these cases the analysis of these structures can be related to that of an AFP cavity, thus allowing the use of the proposed extended analysis, implementation of semiconductor-based AFP cavities offers the advantage of a well-known, reliable fabrication process based on cost-effective vertical cavity technology.

II. DEVICE DESCRIPTION AND PRINCIPLE

The reflectivity of an AFP cavity, under normal incidence, can be described by the following equation:

\[ R = \frac{r_f - r_b e^{-ikd}}{1 - r_f r_b e^{-ikd}} \]  \hspace{1cm} (1)

Being \( r_f \) the front (i.e., the input) mirror amplitude reflectivity, \( r_b \) the back mirror amplitude reflectivity, \( k = 2\pi/\lambda \) the input field wavenumber (with \( \lambda \) the in-vacuum wavelength), and \( d \) the physical length of the cavity, that for simplicity is assumed to be completely filled with a...
nonlinear medium whose complex refractive index \( n \) is given by:

\[
n = n_0 + \frac{n_2 P_c / P_{sat}}{1 + P_c / P_{sat}} - \frac{i \lambda}{2 \pi} \left( \frac{\alpha_n}{1 + P_c / P_{sat}} + \frac{\alpha_0}{1 + P_c / P_{sat}} \right) \quad (2)
\]

In (2), \( n_0 \) and \( n_2 \) are the linear and non linear components associated to the real part of the refractive index, respectively, whereas the terms proportional to \( \alpha_n \) and \( \alpha_0 \) represent the linear and non linear components of the imaginary part of \( n \) (i.e. the non-saturable and the small-signal nonlinear absorption coefficients of the medium), respectively. As seen from (2) both the imaginary and real parts of the nonlinear components of \( n \) are saturable, and depend on the value of the optical power inside the cavity \( P_c \) and the saturation power \( P_{sat} \). The real and imaginary parts of the nonlinear components of the complex refractive index are related through the Kramers-Kronig relations. By putting (2) into (1) the spectral reflectivity of the nonlinear AFP cavity can be computed, as a function of \( P_c \), which is related to the input intensity \( P_{in} \) from:

\[
P_c = \frac{(1 - r_f^2) P_{in}}{1 - r_f n_0 e^{-i kd}}.
\]

Thus, the effect of varying the optical power level inside the resonator is to modulate the optical path and the losses of the cavity. In turn, the optical path modulation produce a shift of the cavity resonances, i.e. the wavelengths for which the round-trip phase shift \( \phi_n \) satisfies the following condition:

\[
\phi_n = 2k \cdot \text{Re}\{n\}, \quad d = 2\pi.
\]

The nonlinear modulation of the (real) refractive index can produce a maximum variation \( \Delta \) (for \( P_c >> P_s \)) of the round-trip phase given by:

\[
\Delta = 2n_k d l.
\]

The dispersive bistable behavior can be explained by observing that the change of resonance produce a change of the power inside the cavity which, under proper condition, can enhance the resonance conditions. This positive feedback loop leads to a regime of instability, resulting in bistability between input and output powers. On the other hand, the change of cavity losses produces a change in the depth of the resonances. Absorptive bistability is explained by noting that when the input intensity is increased the internal intensity is also increased, lowering the absorption and thus producing a further increase of the cavity internal intensity. If the incident field is subsequently lowered, the field inside the cavity tends to remain large because the absorption of the medium has already been decreased. As an example Fig. 1 shows the effect of resonance shift and amplitude variation for different values of intracavity power. For the structure of Fig. 1 the cavity impedance is matched (i.e. the reflectivity value expressed by (1) is ideally zero at resonance) for weak, non saturating values of the input field. This condition means that the following relationship is satisfied:

\[
\sqrt{R_f} = \sqrt{R_b} e^{-i(\alpha_n + \alpha_0) d}.
\]

Where \( R_f \) (\( R_b \)) is the front (back) mirror intensity reflectivity (\( R_{fb} = |r_f|^2 \)).

However, as pointed out in previous works, in order to have an efficient bistable behavior a large nonlinearity is desirable (both dispersive and absorptive), in conjunction with a sufficiently high value of the cavity finesse, providing the required feedback. This means that in practical cases, for effective bistable operation, the top mirror reflectivity satisfies the following relation:

\[
\sqrt{R_f} > \sqrt{R_b} e^{-i(\alpha_n + \alpha_0) d}.
\]

When the inequality (7) is satisfied, the cavity impedance is matched for a cavity field partially
saturating the absorption value \([10]\). This is shown in Fig. 2, where the spectral reflectivity as a function of intracavity power is plotted for top mirror amplitude reflectivity satisfying (7). The intracavity power \(P_{IM}\) for which the cavity impedance is matched at resonance can be calculated from (1) to be:

\[
P_{IM} = P_{sat} \left[ \frac{\alpha_d}{\ln(\left| r_f \right| / \left| r_t \right|)} - 1 \right] .
\]

The plots of Fig. 2, suggest that in order to implement an efficient, high-contrast bistable device the incident field wavelength should be tuned at the values assumed by the resonance under impedance matching condition. In all the following simulation results we satisfied this condition. We will also assume an ideal back mirror with reflectivity \(R_b\) of 100%.

III. BISTABILITY ANALYSIS

Fig. 3 shows the reflected power \(P_r\) as a function of \(P_{in}\) for different values of the small-signal saturable losses \(\alpha_{sd}\), and fixed values of \(R_f = 0.9\), and \(\Delta = 0.0125\). It can be seen that, as expected, wider bistability is obtained for increased absorptive nonlinearity.

![Fig.3. Normalized output reflected power vs. incident power for different values of \(\alpha_{sd}\), and \(R_f = 0.9\), \(\Delta = 0.0125\), \(\alpha_{sd} = 0.025\).](image)

The effect of increasing the dispersive nonlinearity is shown in Fig. 4 where similar plots are displayed with the same values of the parameters as in Fig. 3 except that now \(\Delta = 0.025\). For this larger nonlinear phase change, the device exhibits bistable behaviour also starting from the lowest value of the small-signal saturable absorption. However, it can be seen from the curves of both Fig. 3 and Fig. 4 that in some cases the impedance matching condition (i.e. the minimum of \(P_{in}\) is obtained for values of \(P_{in}\) that lies outside or on the edge of the bistable region. For an optimized device, exhibiting the maximum contrast between the two stable states, the incident power producing the minimum of reflectivity (i.e. impedance matching) should fall within the bistable region (like in the curve associated with \(\alpha_{sd} = 5\) in Fig. 3, or that associated to \(\alpha_{sd} = 1\) or \(\alpha_{sd} = 2.5\) in Fig. 4). Furthermore, the power level of the other possible stable state corresponding to the impedance-matching power should be sufficiently high to guarantee a fair throughput of the device. One should then try to optimize the different cavity parameters for the desired target design.

The effect of front mirror reflectivity \(R_f\) was also investigated, and the results are shown in Fig. 5 for a value of \(\Delta = 0.025\), \(\alpha_{sd} = 2.5\) and \(\alpha_{sd} = 0.025\). Although increasing the value of the front mirror reflectivity, and hence the feedback of the system, produce a wider bistable region, it can be also seen that for the largest value of \(R_f\) pushes the impedance matching point at very high level of the input power (see (5)), outside the bistable region. As said before this situation would limit the optimum contrast ratio between the two stable states. Furthermore, in order to realize practical compact devices one should try to get an optimized design for values of refractive nonlinearity not too large.

![Fig.5. Normalized output reflected power vs. incident power for different values of \(R_f\); \(\Delta = 0.025\), \(\alpha_{sd} = 2.5\), \(\alpha_{sd} = 0.025\).](image)

Fig. 6 shows the nonlinear reflected power as a function of incident power for different values of \(R_f\) and a value of \(\Delta = 0.0125\). The value of the small-signal saturable loss was maintained to \(\alpha_{sd} = 2.5\). However, in order to compensate for the smaller dispersive nonlinearity, the value of non-saturable losses was reduced to \(\alpha_{sd} = 0.01\). The effect of lower non saturable losses is indeed that of reducing the threshold for bistability [11]. Furthermore, in presence of lower non-saturable losses the impedance matching incident power is pulled back to within the bistable region also for the highest investigated value of front mirror reflectivity.
Fig. 6. Normalized output reflected power vs. incident power for different values of $R_f$: $\Delta = 0.0125$, $\alpha_{d} = 2.5$, $\alpha_{ns} = 0.01$.

IV. CONCLUSION

In conclusion we have presented a spectral model for investigating and optimizing the operation of a bistable device based on an AFP cavity filled with a dispersive and absorptive nonlinear medium. We perform a number of simulations in order to inspect the effect of the various cavity parameters on the device characteristic. The results show that, although it would be relatively easy in principle to have bistable behavior in such kind of nonlinear cavities, for optimized operation and practical values of the nonlinear coefficients, care must be taken in choosing the proper nonlinear materials and the different cavity parameters.

REFERENCES