Scuola Superiore Sant'Anna di Studi Universitari e di Perfezionamento

The Migration Toward the Optical Internet

Lesson 7

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Introduction to Mathematical Programming

- Def. Decision variables
  - variables whose values are to be decided in some optimal fashion
  - x_j, j=1, 2, …, n
- Def. Objective function
  - a combination of coefficients and decision variables (mathematical programming)
  - $Z=f(x_1, x_2, \ldots, x_n)$
  - a linear combination of the decision variables (linear programming)
  - $Z=c_1 x_1 + c_2 x_2 + \ldots + c_n x_n$

Introduction to Mathematical Programming (2)

- Def. Constraints
  - equalities and inequalities with combination of the decision variables that limit the values the decision variables can take (mathematical programming)
  - equalities and inequalities with linear combination of the decision variables (linear programming)

$$a_1 x_1 + a_2 x_2 + \ldots + a_n x_n \begin{Bmatrix} \leq \\ = \\ \geq \end{Bmatrix} b$$

Introduction to Mathematical Programming (3)

- The objective of a mathematical program is usually to minimize or maximize the objective function
  - To convert a maximization to a minimization is sufficient to change the sign of the objective function
  - Max Z= Min -Z
- Def. Solution
  - A proposal of specific values for the decision variables
- Def. Feasible Solution
  - A solution $(x_1, x_2, \ldots, x_n)$ is called feasible if it satisfies all of the constraints
- Def. Feasible Solution Space
  - collection of all the solutions that satisfies the limitation imposed by the constraints
- Def. Optimal Solution
  - the feasible solution that maximizes or minimizes the objective function
- Def. Globally Optimal Solution
  - it is an optimal solution with respect to the entire feasible solution space
- Def. Locally Optimal Solution
  - it is an optimal solution with respect to only a limited portion of the feasible solution space

Introduction to Mathematical Programming (4)

- Mathematical programs can often be solved with algorithms
- Def. Algorithm
  - solution procedure that consists of a number of understandable steps and can be implemented on a computer
- Some algorithms for some problems are exact and will be guaranteed to always produce a globally optimal solution
- Sometimes an algorithm to produce a globally optimal solution is not known
- Def. Heuristic algorithms
  - algorithms that use intuitive procedures to develop solutions that may be close to being optimal (i.e., sub-optimal solutions)

Linear Programming

- Def. Linear Program
  - a mathematical program in which both the objective function and and the constraint equations are linear function of the decision variables
- Nonlinear Programming deals with the solution of nonlinear models

Linear Programming Standard (Canonical) Form

- All inequalities are less-thans
- All decision variables are non-negative

Objective function

$$\max Z = c_1 x_1 + c_2 x_2 + \ldots + c_n x_n$$

Subject to (Constraints)

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \leq b_1$$

$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \leq b_2$$

$$\vdots$$

$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \leq b_m$$

$$x_1, x_2, \ldots, x_n \geq 0$$

Linear Programming Matrix Representation

$$\min Z = \mathbf{c}^{\mathrm{T}}\mathbf{x}$$

$$\mathbf{A}\mathbf{x} = \mathbf{b}$$

$$\mathbf{x} \geq \mathbf{0}$$

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

$$\mathbf{c} = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix} \qquad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \qquad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

Constraint Conversion

- Conversion of an inequality constraint to an equality constraint
- Introduction of a *slack variable*

$$a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{in}x_n \leq b_i$$

$$\Downarrow$$

$$a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{in}x_n + w_i = b_i$$

$$w \geq 0$$

Constraint Conversion (2)

- Conversion of an inequality constraint to an equality constraint
- Introduction of a *surplus variable*

$$a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{in}x_n \geq b_i$$

$$\Downarrow$$

$$a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{in}x_n - y_i = b_i$$

$$y_i \geq 0$$

Constraint Conversion (3)

- Conversion of an equality constraint to two inequality constraints

$$a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{in}x_n = b_i$$

$$\Downarrow$$

$$a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{in}x_n \leq b_i$$

$$a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{in}x_n \geq b_i$$

Linear Programming Examples

- Infeasible problem
  - If a problem has no feasible solution is called infeasible
  - the second constraint implies that $x_1+x_2 \geq 4.5$ which contradicts the first constraint

$$
\begin{aligned}
&\max 5x_1 + 4x_2 \\
&s.t. \\
&x_1 + x_2 \leq 2 \\
&-2x_1 - 2x_2 \leq -9 \\
&x_1, x_2 \geq 0
\end{aligned}
$$

Linear Programming Examples (2)

- Unbounded problem
  - A problem is unbounded if it has feasible solutions with arbitrarily large objective values
  - if x_2 is set to zero and x_1 is let be be arbitrarily large as long as $x_1>2$
    - the solution is feasible
    - and the objective function gets large too

$$
\begin{aligned}
&\max x_1 - 4x_2 \\
&s.t. \\
&-2x_1 + x_2 \leq -1 \\
&-x_1 - 2x_2 \leq -2 \\
&x_1, x_2 \geq 0
\end{aligned}
$$

The Min Cost Flow Problem

- Given N nodes connected with weighted arcs (links)
- Each node is either a source or a sink of flow
- Amount of flow originating at a node
  - $b_i>0$, source
- Amount of flow terminating at a node
  - $b_i<0$, sink
- All the flow generated must be consumed
  - $\Sigma_{i=1}^{N} b_i=0$
- c_{ij}=cost of a unit of flow traversing link (i,j)
- x_{ij}=amount of flow between node i and node j
- Objective:
  - route all the flow generated to the destinations with minimum cost

The Min Cost Flow Problem (2)

- The first constraint represents the flow conservation constraint

$$\min Z = \sum_{i,j} c_{ij} x_{ij}$$

$$s.t.$$

$$\sum_{j=1}^{N} x_{ij} - \sum_{k=1}^{N} x_{ki} = b_i \quad i = 1,2,\ldots,N$$

$$x_{ij} \geq 0 \quad i, j = 1,2,\ldots,N$$

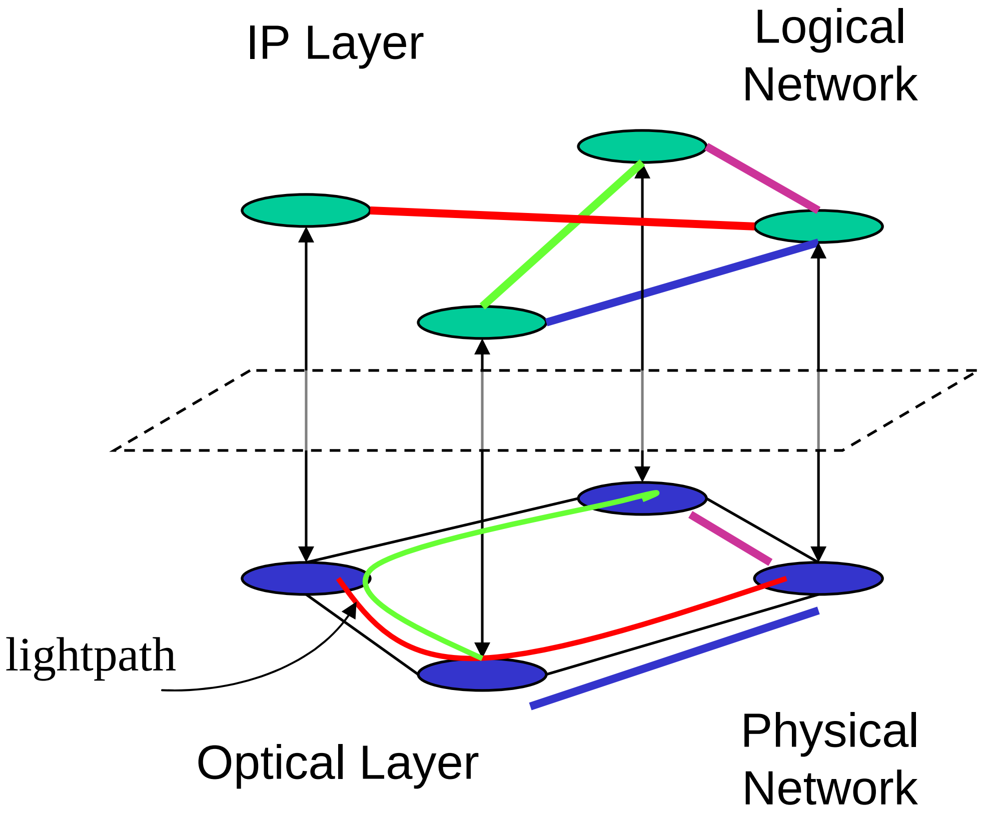
Optical Network Design

- The Optical Layer provides high-speed circuit switched connections, or lightpaths, between pairs of higher layer equipments (e.g., SONET/SDH muxes, IP routers, ATM switches)
- A network using lightpaths is called a wavelength routing network
- Optical Network Design Problem:
  – How to design Optical Layer to satisfy requirements of higher-layer SONET/SDH or IP network

Optical Network Design (2)

- Consider the Optical Network Design Problem when only IP and DWDM layer are involved
- The topology seen by IP routers is the topology provided by the optical layer
  - IP router topology → *lightpath topology* (often called the *logical* or *virtual topology*)
  - OL topology → *physical topology*

Logical and Physical Topology



Wavelength-Routing Network Design

- Given:
  - physical (fiber) topology
  - traffic requirements (traffic matrix)
- Problems
  1. Design a logical (lightpath) topology that interconnects the IP routers → Logical (Lightpath) Topology Design (LTD) problem
  2. Realize the topology within the optical layer → Routing and Wavelength Assignment (RWA) problem

Grooming Problem

- The term *grooming* is commonly used to refer to the packing of low-speed SONET/SDH circuits into higher speed circuits
- Conceptually IP routers too can be considered to provide the grooming function at the packet level
- In order to exploit the benefits of lightpaths higher-layer traffic must be groomed appropriately

LTD and RWA Problems

- Jointly solving the wavelength-routing network design problem, is complicated
- General approach is to divide the wavelenth-routing network design problem into the LTD problem and the RWA problem

Lightpath Topology Design (LTD) Problem

- Assumptions for considered LTD problem
  - no constraints imposed by the optical layer (e.g., no limit on the lightpath length, no limit on number of lightpath traversing a link)
  - All lightpaths are bidirectional
    - If a lightpath from node i to node j is used, a lightpath from node j to node i is used too

LTD Problem Mathematical Formulation

- Decision variables
  - λ_{ij}^{sd}: fraction of the traffic between source-destination pair *(s,d)* routed over logical topology link *(i,j)*
  - b_{ij}=1 if the designed logical topology has a link (lightpath in the physical topology) between node *i* and node *j*; b_{ij}=0 otherwise
- Constants
  - λ^{sd}: average arrival rate for packets for source-destination (s-d) pair *(s,d)*
  - Δ: maximum number of ports in a single IP router
    - This indirectly constraints the cost of the IP routers
    - This also constraints the number of lightapths in the network to n Δ, where n is the number of nodes in the network, since each lightpath starts and ends at an IP router port

LTD Problem Mathematical Formulation (2)

- Other support variables
  - $\lambda_{ij}=\Sigma_{sd}\ \lambda_{ij}^{sd}$: total traffic from all s-d pairs that is routed over logical link *(i,j)*
  - $\lambda_{max}=max_{ij}\ \lambda_{ij}$: congestion over logical link *(i,j)*
    - Assuming that
      - packet arrivals follow a Poisson process
      - packet transmission times are exponentially distributed with mean time given by $1/\mu$
      - the traffic offered to a logical topology link (lightpath) in the network is independent of the traffic offered to other links, each link can be modeled as a M/M/1 queue
    - The average queueing delay on link *(i,j)* can be computed as
      - $d_{ij}=1/(\mu - \lambda_{ij})$
    - The throughput can be defined as the minimum value of the offered load for which the delay on any link becomes infinite
      - $\lambda_{max}=\mu$

LTD Problem Mathematical Formulation (3)

- Objective function
  - $min\ \lambda_{max}$
  - The objective is to minimize the maximum congestion along the logical network links
- Subject to:

LTD Problem Mathematical Formulation Constraints

- Flow conservation constraint
  - the packets to be routed between each s-d pair with the *flow* of a commodity
  - left-hand side of the flow conservation constraint at node *i* computes the net flow out of node *i* for one commodity *(sd)*
  - net flow is the difference between the outgoing flow and the incoming flow
  - right-hand side is 0 if node *i* neither the source nor the destination for that commodity ($i \neq s, d$)
    - If node *i* is the source of the flow ($i=s$) $\Rightarrow$ net flow equals λ^{sd}
    - If node *i* is the destination ($i=d$) $\Rightarrow$ net flow equals - λ^{sd}

$$\sum_j \lambda_{ij}^{sd} - \sum_j \lambda_{ji}^{sd} = \begin{cases} \lambda^{sd} & \text{if } s = i \\ -\lambda^{sd} & \text{if } d = i \\ 0 & \text{otherwise} \end{cases} \quad \forall s, d, i$$

LTD Problem Mathematical Formulation Constraints (2)

- Total flow on a logical link
  - $\lambda_{ij}=\Sigma_{sd}\ \lambda_{ij}^{sd}\ \ \forall\, i,j$
    - definition of λ_{ij}: total traffic from all s-d pairs that is routed over logical link *(i,j)*
  - $\lambda_{ij} \leq \lambda_{max}\ \forall\, i,j$
    - together with the fact that the objective function is minimimzing λ_{max} ensures that the minimum value of λ_{max} is the congestion
  - $\lambda_{ij}^{sd} \leq \mathrm{b}_{\mathrm{ij}}\ \lambda^{sd}\ \ \forall\, i,j,s,d$
    - If the link (*i,j*) exists in the logical topology (b_{ij}=1) there is flow along the link
    - otherwise there cannot be any flow along link *(i,j)* because it does not exists)

LTD Problem Mathematical Formulation Constraints (3)

- Degree constraints
  - $\Sigma_i \, b_{ij} \leq \Delta \;\; \forall j$
  - $\Sigma_j \, b_{ij} \leq \Delta \;\; \forall i$
    - Ensure that the designed topology has no more than Δ links into and out of each node
- Bidirectional lightpath constraint
  - $b_{ij}=b_{ji} \quad \forall i,j$
    - Ensures that the resulting logical topology has only bidirectional lightpaths

LTD Problem Mathematical Formulation Constraints (4)

- Nonnegativity and integer constraints
  - $\lambda_{ij}^{sd}, \lambda_{ij}, \lambda_{max} \geq 0 \quad \forall i, j, s, d$
  - $b_{ij} \in \{0, 1\} \;\; \forall i, j$
  - Ensures that all the variables take positive values
  - b_{ij} can only take binary values
- The resulting problem is a Mixed Integer Linear Program (MILP) because it involves both real and binary variables
- The LTD problem mathematical programming formulation is calle a LTD-MILP problem

LTD-MILP Solution

- In general Integer Linear Program (ILP) and MILP are NP-hard problem
- NP= Non-deterministic polynomial, the complexity of the problem solution cannot be deterministically expressed with polynomial expression
- Commercial packages are readily available to solve Linear Program, ILP, and MILP and in many cases they are of a larger package of mathematical and/or optimization routines
- Due to the huge amount of time that MILP might require to be solved, heuristic techniques are developed to obtain a suboptimal solutions of the problem in less time

Routing and Wavelength Assignment (RWA) Problem

- Definition RWA problem
  - Given a physical network topology and a set of end-to-end lightpath requests (which could be obtained, for example, by solving the LTD problem), determine a *route* and a *wavelength(s)* for the requests, using the minimum possible number of wavelengths
  - Sometimes the two RWA can be further decomposed in two problems
    - Routing
    - Wavelength Assignment

Wavelength Assignment (WA) Problem

- If the routing is already given the WA problem must obey the following constraints
  - Two lightpaths must not be assigned the same wavelength on a given link
  - If no wavelength conversion is available, then a lightpath must be assigned the same wavelength on all the links in its route (*wavelength continuity constraint*)

Static RWA

$$\text{Minimize } F_{\max}$$

such that

$$F_{\max} \geq \sum_{s,d,w} F_{ij}^{sdw} \quad \forall i, j$$

$$\sum_i F_{ij}^{sdw} - \sum_k F_{jk}^{sdw} = \begin{cases} -\lambda_{sdw} & \text{if } s = j \\ \lambda_{sdw} & \text{if } d = j \\ 0 & \text{otherwise} \end{cases} \quad \forall j, s, d, w$$

$$\sum_w \lambda_{sdw} = \Lambda_{sd} \quad \forall s, d$$

$$\lambda_{sdw} = \{0,1\} \quad \forall s, d, w$$

$$F_{ij}^{sdw} = \{0,1\} \quad \forall s, d, i, j, w$$

$$\sum_{s,d} F_{ij}^{sdw} \leq 1 \quad \forall i, j, w$$

Static RWA (2)

- RWA can be formulated as an Integer Linear Program (ILP)
- The objective function is to minimize the flow in each link, which, in turn, corresponds to minimizing the number of lightpaths passing through a particular link
- λ_{sdw}: the traffic (number of connection requests) from any source s to any destination d on any wavelength w
- $\lambda_{sdw} \leq 1$: two or more lightpaths may be set up between the same source-destination pair but they must employ a different wavelength
- $F_{ij}^{sdw} \leq 1$: the traffic (number of connection requests) from source s to destination d on link (i,j) and wavelength w must be assigned to only one path
- Λ_{sd}: number of connection requests source s and destination d

Static RWA with Wavelength Conversion

$$\text{Minimize } F_{\max}$$

such that

$$F_{\max} \geq \sum_{s,d} F_{ij}^{sd} \quad \forall i, j$$

$$\sum_i F_{ij}^{sd} - \sum_k F_{jk}^{sd} = \begin{cases} -\lambda_{sd} & \text{if } s = j \\ \lambda_{sd} & \text{if } d = j \\ 0 & \text{otherwise} \end{cases} \quad \forall i, s, d$$

$$F_{ij}^{sd} \geq 0 \qquad \forall i, j, s, d$$

Static RWA with Wavelength Conversion (2)

- λ_{sd}: traffic (number of connection requests) from any source s to any destination d
- F_{ij}^{sd}: traffic (number of connection requests) from source s to destination d on link (i,j)
- Multicommodity flow formulation
- In many cases full wavelength converison in the network may not be preferred and may not even be necessary due to high cost and limited performance gain
- It is possible that
  - a subset of the nodes allows wavelength conversion
  - a wavelength converter is shared by more than one fiber link
  - a node employs converters that can only convert to a limited range of wavelengths

Routing + WA Problems

- RWA problem is a hard problem
- RWA can be simplified by decoupling the problem into two separate subproblems
  - Routing subproblem
  - Wavelength Assignment (WA) subproblem

Fixed Routing

- Most straightforward approach to routing a connection is to always choose the same fixed route for a give –source-destination pair
- Example
  - fixed shortest path routing: the shortest-path route for each source-destination pair is calculated off-line using standard shortest path algorithms (e.g., Dijkstra's, Bellmann-Ford)
  - any connection between the specified pair of nodes is established using the pre-determined route
  - Advantage
    - approach for routing connection very simple
  - Drawback
    - if resources (i.e., wavelengths) along the path are scarce
      - it can potentially lead to high blocking in the dynamic RWA
      - large number of wavelengths used in the static RWA
    - fixed routing may be unable to handle fault situations in which one or more links in the network fail

Fixed-Alternate Routing

- It considers multiple routes between the same source-destination pair
- In fixed-alternate routing, each node in the network maintains a routing table that contains an ordered list of a number of fixed routes to each destination node
- For example the routes may include the shortest-path route, the second shortest-path route, the third shortest-path route, etc.
- A primary route between a source node *s* and a destination node *d* is defined as the first route in the list of routes to node *d* in the routing table at node *s*
- An alternate route between *s* and *d* is any route that does not share any links (is link-disjoint) with the first route in the routing table at *s*
  - The term "alternate routes" is also used to describe all routes (including the primary route) from s to d

Fixed-Alternate Routing (2)

- When a connection request arrives, the source node attempts to establish the connection on each of the routes from the routing table in sequence until a route with a valid wavelength assignment is found
- If no available route is found from the list of alternate route, then the connection request is blocked and lost
- Advantages
  - Fixed-alternate routing provides simplicity of control for setting up and tearing down lightpaths
  - It may also be used to provide some degree of fault tolerance upon link failures
  - Alternate routing can significantly reduce the connection blocking probability compared to fixed routing
  - For certain networks having as few as two alternate routes provides significantly lower blocking probability than having full wavelength conversion at each node with fixed routing

Adaptive Routing

- In adaptive routing the route from a source node to a destination node is chosen dynamically, depending on the network state
- The network state is determined by the set of all connections that are currently in progress

Adaptive Routing (2)

- One form of adaptive routing is adaptive shortest-cost-path routing (suited for wavelength converted networks)
  - each unused link in the network has a cost of 1
  - each used link in the network has a cost of ∞
  - each wavelength converter link has a cost of c units
  - if wavelength conversion is not available $c = \infty$
- When a connection arrives, the shortest-cost path between the source node and the destination node is determined
- By choosing the wavelength-conversion cost appropriately it can be ensured that wavelength-converted routes are chosen only when wavelength continuous paths are not available
- A connection is blocked only when there is no route (either wavelength-continuous or wavelength converted) from source to destination

Adaptive Routing (3)

- Adaptive routing requires extensive support from the from the network control and management protocols to continuously update the routing tables and the network state at the nodes
- Advantages:
  – adpative routing results in lower connection blocking than fixed and fixed-alternate routing

Adaptive Routing (4)

- Least Congested Path (LCP) routing
- Similarly to alternate routing for each source-destination pair a sequence of routes is pre-selected
- Upon the arrival of a connection request the least-congested path among the pre-determined routesis chosen
- The congestion on a link is measured by the number of wavelengths available on the link
- Links with fewer available wavelengths are considered more congested
- The congestion on a path is indicated by the congestion in on the most congested link in the path
- Disadvantages
  - computational complexity
    - in choosing the least-congested path, all links in all candidate path have to be examined

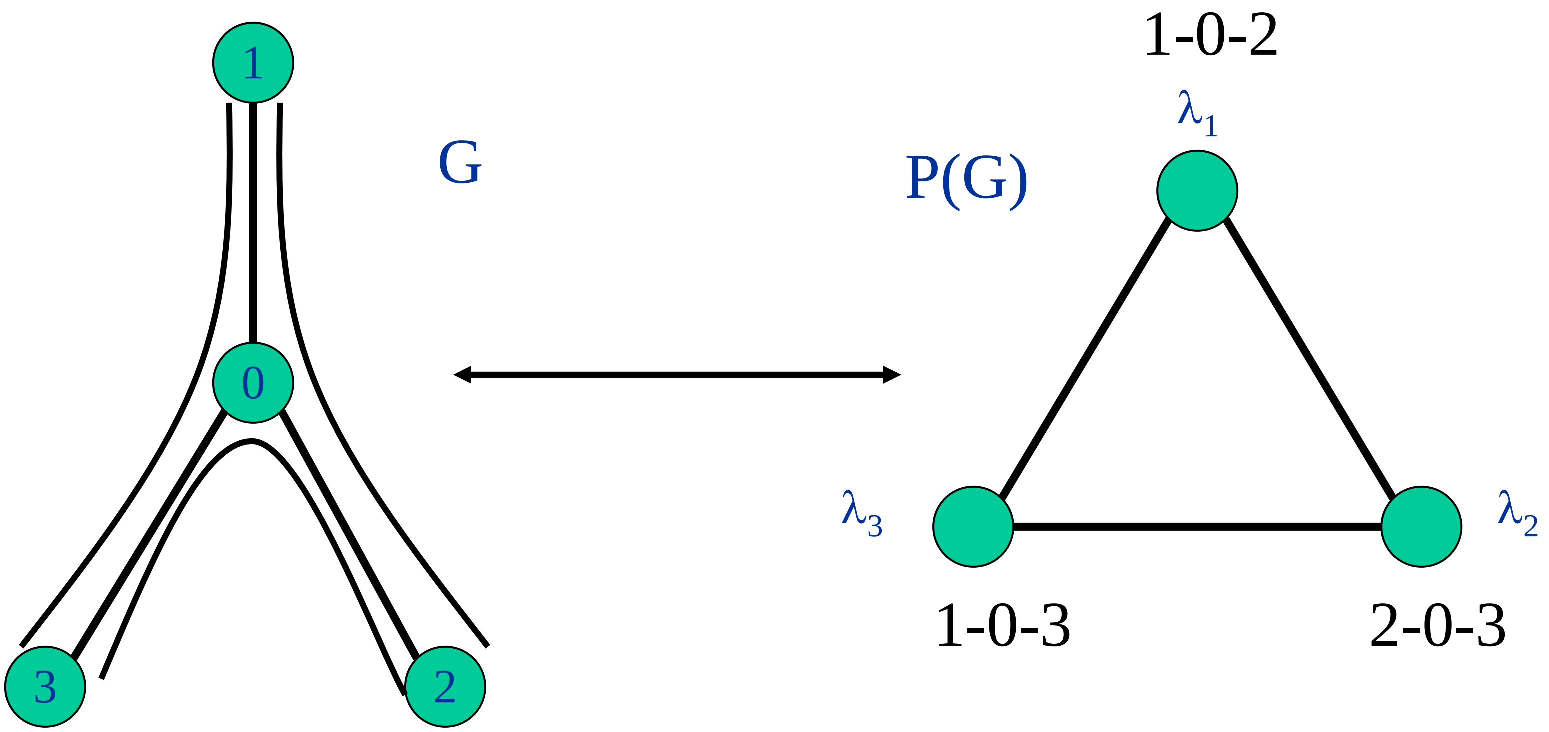
Wavelength Assignment

- It can be distinguished in
  - Static Wavelength Assignment
    - given a fixed set of lightpaths and their routes assign a wavelength to each lightpath such that no two lightpaths share the same wavelength on a given fiber link and the number of wavelengths used is minimized
  - Dynamic Wavelength Assignment
    - lightpaths requests arrive one at a time (either incremental or dynamic traffic)

Static Wavelength Assignment

- Assigning wavelengths to different lightpaths in a manner that minimizes the number of wavelengths used under the wavelength-continuity constraint reduces to the graph-coloring problem
- Graph coloring problem:
  - Construct and auxiliary graph P(G) (path graph of G) such that each lightpath in the system is represented by a node in graph P. There is an undirected edge between two nodes in graph P if the corresponding lightpaths pass through a common physical fiber link
  - Color the nodes of the graph P such that no two adjacent nodes have the same color
  - The minimum number of colors needed to color the nodes of the auxiliary graph in this manner is called chromatic number of the graph
  - The minimum number of wavelengths required to solve to solve the WA problem is the chromatic number of P(G)

Auxiliary Graph Construction Example



Graph Coloring Solution

- Graph coloring problem has been shown to be NP-complete
- However there are efficient sequential graph-coloring algorithms that are optimal in the number of colors used
- In a sequential graph-coloring approach
  - vertices are sequentially added to the portion of the the graph already colored
  - new colorings are determined to include each newly adjoined vertex
  - at each step the the total number of colors necessary is kept to a minimum
- The optimal solution depends on the node ordering utilized
- So,me particular sequential vertex coloring yield the graph P chromatic number

Dynamic Wavelength Assignment

- For the dynamic case heuristic methods must be used to assign wavelength to lightpaths
- For the dynamic problem, instead of attempting to minimize the number of wavelengths as in the static case, the number of wavelengths is assumed fixed and the objective is to minimize connection blocking

Proposed Heuristics

- Random Wavelength Assignment (R)
  - First it determines the set of wavelengths available among all the possible wavelengths
  - Among the available wavelengths one is chosen randomly (usually with uniform probability)

Proposed Heuristic (2)

- First-Fit (FF)
  - All wavelengths are numbered
  - When searching for available wavelengths a lower numbered wavelength is considered before a higher-numbered wavelength
  - The first available wavelength is then selected
  - Advantages
    - It requires no global information $\Rightarrow$ no communication overhead
    - Lower computational cost than Random because no need to search the entire wavelength space for each route
  - Idea behind FF
    - packing all of the in-use wavelengths toward the lower end of the wavelength space so that continuous longer path toward the lower end of the wavelength space will have higher probability of being available

Proposed Heuristic (3)

- Least-Used (LU)/SPREAD
  - LU selects the least used wavelength in the network
  - Idea behind LU
    - balancing the load among all the wavelengths
  - LU breaks the long wavelength path quickly
  - Only connection requests that traverse a small number of links will be serviced in the network
  - Disadvantages
    - Global informartion required to compute the least-used wavelength
    - Introduces additional communication overhead
    - It requires additional storage and computation cost

Proposed Heuristic (4)

- Most-Used (MU)/Pack
  - MU attempts to select the most-used wavelength in the network (opposite of LU)
  - Communication overhead, storage, and computation cost similar to LU
  - Advantages
    - MU outperforms LU
    - MU slightly outperforms FF

Proposed Heuristic (5)

- Min-Product (MP)
  - Used in multi-fiber networks
  - In a single-fiber network MP becomes FF
  - Idea
    - Packing wavelengths into fibers minimizing the number of fibers in the network
  - Drawbacks
    - MP does not perform as well as FF multifiber version
    - MP introduces additional computational cost

Proposed Heuristic (6)

- Least-Loaded (LL)
  - Like MP designed for multi-fiber networks
  - Idea
    - selecting the wavelength that has the largest residual capacity on the most-loaded link along a route
    - LL reduce to FF in single-fiber networks
      - when used in single-fiber networks the residual capacity is either 1 or 0 $\Rightarrow$ the lowest-indexed wavelength with residual capacity 1 is chosen

Proposed Heuristic (7)

- Max-Sum (MΣ)
  - Proposed for multi-fiber networks, it can be also applied to single-fiber networks
  - Idea
    - MΣ consider all possible paths (lightpaths with their preselected routes) in the network and attempts to maximize the remaining path capacities after lightpath establishment
  - MΣ assumes that
    - the set of possible connection requests is known in advance
    - the route for each connection is preselected
    - this requirements can be achieved since the traffic matrix is assumed to stable for a period of time and routes can then be computed for each potential path on the fly

Proposed Heuristic (8)

– MΣ algorithm overview
  - MΣ chooses the wavelength that minimizes the capacity loss on all lightpaths
  - The lightpath capacity on a wavelength is the number of fibers on which wavelength j is available on the most-congested link along the path
  - The lightpath capacity is defined as the sum of lightpath capacity on all wavelengths
  - The capacity loss is defined as the difference between the lightpath capacity before the lightpath is set up and after the lightpath is set up

Proposed Heuristic (9)

- Relative Capacity Loss
  - RCL is based on MΣ
  - MΣ can be viewed as an approach that chooses the wavelength that minimizes the capacity loss on all lightpaths
  - the lightpath is set up and after the lightpath is set up
  - Idea
    - RCL is based on the observation that minimizing total capacity loss sometimes does not lead to the best choice of wavelength
    - RCL calculates the Relative Capacity Loss for each path in the on each available wavelength and then chooses the wavelength that minimizes the sum of the relative capacity loss on all the paths

Proposed Heuristic (10)

- Objective of all previous WA scheme was to minimize blocking probability
- In general longer lightpaths have higher probability of getting blocked than shorter ones
- Wavelength Reservation (Rsv) and Protecting Threshold (Thr) schemes attempt to protect longer paths
  - Rsv and Thr characteristics:
    - They do not specify which wavelength to choose but they specify whether or not the connection request can be assigned a wavelength under the current wavelength-usage conditions $\Rightarrow$ they must be combined with with other WA schemes
    - Rsv and Thr attempt to protect only the connections that traverse multiple fiber links (multihop connections)
- For these schemes overall blocking probability may be higher but greater degree of fairness is achieved

Proposed Heuristic (11)

- Wavelength Reservation (Rsv)
  - A given wavelength on a specified link is reserved for a particular traffic stream between a source-destination pair
  - Other traffic streams are not allowed to utilized that reserved wavelength even if the reserved wavelength is idle

Proposed Heuristic (12)

- Protection Threshold (Thr)
  - In Thr a single hop connection is assigned a wavelength only if the the number of idle wavelengths on the link is at or above a given threshold

Joint LTD and RWA Formulation (1)

- Definitions
  - s and d are denote source and destination of a packet respectively
  - i and j denote originating and terminating node, respectively, in a lightpath
  - m and n denote endpoints of a physical link
  - N : number of nodes in the network
  - M : maximum number of wavelengths per fiber
  - P_{mn}: physical topology; $P_{mn} = P_{nm} = 1$ if and only if (iff) there exists a direct physical fiber link between nodes m and n; $P_{mn} = P_{nm} = 0$, otherwise (i.e., fiber links are assumed to be bidirectional)

Joint LTD and RWA Formulation (2)

- d_{mn} : fiber distance from node *m* to node *n*
  - for simplicity in expressing packet delays, d_{mn} is expressed as a propagation delay (in time units)
- $T_i(\geq 1)$: number of transmitters at node *i*
- $R_i(\geq 1)$: number of receivers at node *i*
- λ_{sd} : traffic matrix which denotes the average rate of traffic flow from node *s* to node *d*, with (λ_{ss}=0)
  - packet interarrival time at node *s* and packet lengths (service time) are exponentially distributed so standard M/M/1 queuein results can be applied to each virtual network link (or "hop")
- *C* : capacity of each channel

Joint LTD and RWA Formulation (3)

- Variables
  - V_{ij} : virtual topology
    - V_{ij}=1 if there exists a lightpath from node *i* to node *j* in the virtual topology
    - V_{ij}=0 otherwise
    - Lightpaths are not assumed to be bidirectional: $V_{ij}=1 \Rightarrow V_{ji}=1$
  - λ_{ij}^{sd} : traffic flowing from *s* to *d* and employing V_{ij} as an intermediate virtual link
  - Physical Topology Route
    - p_{mn}^{ij}=1 if the fiber link P_{mn} is present in the lightpath for virtual link V_{ij}
    - p_{mn}^{ij}=0 otherwise
  - Wavelength color
    - c_k^{ij}=1 if a lightpath from originating node *i* to terminating node *j* is assigned to the color *k*, *k*=1, 2, …, M
    - c_k^{ij}=0 otherwise

Joint LTD and RWA Formulation (4)

- Objective

Delay minimization

$$Minimize \sum_{ij}\left[\sum_{sd}\lambda_{ij}^{sd}\left(\sum_{mn} p_{mn}^{ij}\cdot d_{mn}+\frac{1}{C-\sum_{sd}\lambda_{ij}^{sd}}\right)\right]$$

or maximizing the offered load

$$\min\left[\max\left(\sum_{sd}\lambda_{ij}^{sd}\right)\right]\equiv \max\left\{\frac{C}{\min\left[\max\left(\sum_{sd}\lambda_{ij}^{sd}\right)\right]}\right\}\forall i, j$$

Joint LTD and RWA Formulation (5)

- First objective function
  - is nonlinear
  - innermost brackets
    - the first component corresponds to the propagation delays on the links *mn* which form the lightpath *ij*
    - the second component corresponds to delay due to queueing and packet transmission on lightpath *ij* (using a M/M/1 queueing model for each lightpath)
    - if queuing delays are neglected (i.e., second component in the innermost brackets null) the problem is a MILP
- Second objective function
  - is nonlinear
  - it minimizes the amount of traffic flowing through any lightpath
  - this corresponds to obtaining a virtual topology which can minimize the offered load to the network if the traffic matrix is allowed to scale up

Joint LTD and RWA Formulation (5)

- Constraints
  - on virtual topology connection matrix V_{ij}
  - ensure that the number of lightpaths emanating out of and terminating at a node are at most equal to that node's out-degree and in-degree

$$\sum_{j} V_{ij} \leq T_i \quad \forall i$$

$$\sum_{i} V_{ij} \leq R_j \quad \forall j$$

Joint LTD and RWA Formulation (6)

- on physical route variables p_{mn}^{ij}
  - first two equations constrain the problem so that p_{mn}^{ij} can exist only if there is a physical fiber and a corresponding lightpath present
  - last three equations are multicommodity equations that account for the routing of a lightpath from its origin to its destination

$$p_{mn}^{ij} \leq P_{mn}$$

$$p_{mn}^{ij} \leq V_{ij}$$

$$\sum_m p_{mk}^{ij} = \sum_n p_{kn}^{ij} \quad \text{if} \quad k \neq i, j$$

$$\sum_n p_{in}^{ij} = V_{ij}$$

$$\sum_m p_{mj}^{ij} = V_{ij}$$

Joint LTD and RWA Formulation (7)

– on virtual topology traffic variables λ_{ij}^{sd}
  - routing of packet traffic on the virtual topology
  - they take into account the fact that the combined traffic flowing through a channel (lightpath) cannot exceed the channel capacity

$$\lambda_{ij}^{sd} \geq 0$$

$$\sum_{j} \lambda_{sj}^{sd} = \lambda_{sd}$$

$$\sum_{i} \lambda_{id}^{sd} = \lambda_{sd}$$

$$\sum_{i} \lambda_{ik}^{sd} = \sum_{j} \lambda_{kj}^{sd} \quad \text{if} \quad k \neq s, d$$

$$\sum_{s,d} \lambda_{ij}^{sd} \leq V_{ij} \cdot C$$

Joint LTD and RWA Formulation (8)

- on coloring of lightpaths c_k^{ij}
  - each lightpath must be of one color only (wavelength continuity constraint)
  - colors used in different lightpaths are mutually exclusive over a physical link

$$\sum_k c_k^{ij} = V_{ij}$$

$$\sum_{ij} p_{mn}^{ij} \cdot c_k^{ij} \leq 1 \quad \forall m, n, k$$